Recurrence Relations

Reset Progress Reveal Solutions

1 Recursion trees

Consider the recurrence relation $T(n) = 5T(\frac{n}{4}) + 2n$
What is the number of problems in level 4? (We use the convention that the root problem of size n is on level 0.)

- ○ 1024
- ○ 125
- ● 625
- ○ 4096

Correct

What is the size of each problem in level 5?

- ○ $\frac{n}{512}$
- ○ $\frac{n}{3125}$
- ● $\frac{n}{1024}$
- ○ $\frac{n}{625}$

Correct

What is the total contribution in level i?

- ○ $(\frac{4}{5})^i \times 2n$
- ● $(\frac{5}{4})^i \times 2n$
- ○ $(\frac{4}{5})^{(i-1)} \times 2n$
- ○ $(\frac{5}{4})^{(i-1)} \times 2n$

Correct

Which one is true for $T(n)$?

- ○ $T(n) = \Theta(n^{\log_4 5})$
- ○ $T(n) = O(n^2)$
- ○ $T(n) = \Omega(n)$
- ● All of the above!

Correct

2 The master theorem

Remember that the master theorem applies to recurrences of the form

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + O(n^d).$$

Consider the recurrence relation $T(n) = 3T(\frac{n}{81}) + 10\sqrt{\sqrt{n}}$. What are the values of the parameters a, b, d? Write fractional values in the form of $0.x$ or $0.xx$.

$a =$

3

Correct

$b =$

81

Correct

$d =$

0.25

Correct

Which one is true for $T(n)$?

- ○ $T(n) = \Omega(n)$
- ○ $T(n) = \Theta(n^2)$
- ● $T(n) = \Omega(\log(n)\sqrt{\sqrt{n}})$

Correct

3 The substitution method

Consider the recurrence relation $T(n) = 2T(n/2) + 3T(n/3) + n^2$. Which one is the smallest valid bound for $T(n)$?

- ○ $T(n) = O(n)$
- ● $T(n) = O(n^2)$
- ○ $T(n) = O(n^3)$

Correct

Which one would be the best guess to substitute $T(n)$ with if we wanted to prove the above bound? (Which bound would be provable by induction as in the substitution method and is the tightest such bound.)

- ○ $T(n) \leq 12n$
- ○ $T(n) \leq n^3$
- ● $T(n) \leq 6n^2$
- ○ $T(n) \leq 2n^2$

Correct